TURBULENT TRANSITION OF A LAMINAR BOUNDARY LAYER IN THE AXISYMMETRIC FLOW OF A JET OVER PLANE SURFACES NORMAL TO THE STREAM

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Results are presented of experimental investigations of the hydrodynamics of the wall boundary layer on a plate over which an axisymmetric jet flows transversely. The conditions of existence and transition to turbulent of the laminar boundary layer are determined.

In heat transfer calculations it is very important to be able to predict the conditions at which a laminar boundary layer will become turbulent. This is especially important in transverse flow over plane surfaces, particularly of a jet, for which there are practically no experimental data (apart from [1]) on the hydrodynamics of transition.

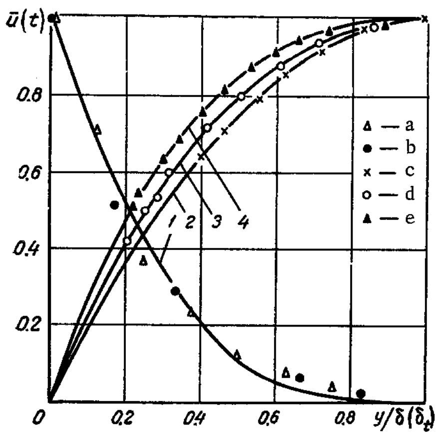

Fig. 1. Temperature and velocity profiles in the critical point region ($0 < \bar{r}_* < 1$) with $u_0 = 15$ m/sec, $\bar{h} = 8$, $\bar{r}_* = 0.4$ (a) and 0.9 (b); $u_0 = 15$ m/sec, $\bar{h} = 2.5$, $\bar{r}_* = 1$ (c, authors' data); $u_0 = 37$ m/sec, $\bar{h} = 0.5$, $\bar{r}_* = 0.83$ (d) and 0.5 (e, data of [1]); 1) calculated from (3); 2–4) calculated from (1).

It was shown in [2] that in transverse flow of an axisymmetric jet over a plate, there are three characteristic flow regions in the wall boundary layer: the critical point region ($0 < \bar{r} < \bar{r}_m$), the transition region ($\bar{r}_m \le \bar{r} \le 2\bar{r}_m$), and the main flow region ($\bar{r} \ge 2\bar{r}_m$).

In the flow of an infinite stream over bodies of different shape, in the immediate vicinity of the critical point, the velocity u_s at the outer edge of the wall boundary layer varies in proportion to the distance from the forward stagnation point, and the boundary layer is laminar. This has been confirmed by numerous experimental investigations. In our case the wall boundary layer in the critical point region is also always laminar, although the oncoming jet is turbulent. The stabilizing factor is the accelerated motion of the fluid particles in the boundary layer due to the negative gradient [2]. The distribution of the velocity and temperature fields within the wall boundary layer in a direction perpendicular to the plate (Fig. 1) confirms the laminar nature of the boundary layer. The velocity profile distribution in the wall boundary layer was determined from measurements of the total head in the boundary layer on the assumption that the static pressure at each section of the boundary layer is constant and equal to that at the surface. The profiles obtained are limited to the value $y/\delta \ge 0.2$, since observations were not made closer to the surface, owing to the finite dimensions of the total pressure tube.

As may be seen from Fig. 1, the velocity profiles in the critical point region (curves 2–4) vary continuously in the presence of pressure gradient, and the variation may be represented by the function [3]

$$u = (2\eta - 2\eta^3 + \eta^4) + \frac{\lambda}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4), \quad (1)$$

where $\eta = y/\delta$ and $\lambda = (\delta^2/\nu)(du_s/dr)$ is a dimensionless quantity which may be computed from a knowledge of the hydrodynamic thickness of the boundary layer [4] as

$$\delta = 1.97 d_0/\mathrm{Re}_0\, a^{1/2}\,(1 - k\bar{r}^{-2})^{1.2},$$

and the velocity at the outer edge of the boundary layer [2]

$$u_s = u_0(a\bar{r} - b\bar{r}^{-3}).$$

Thus,

$$\lambda = 4(a - 3b\bar{r}^{-2})/a(1 - k\bar{r}^{-2})^{2.4}.$$

The quantities a, b, k and $\bar{r}_m$ depend on the dimensionless distance h:

when $\bar{h} \leqslant 6.2$

$$a = 1.5\bar{h}^{-0.22}, \quad b = 0.5\bar{h}^{-0.42},$$

$$k = b/a, \quad \bar{r}_m = \bar{h}^{-0.1};$$

when $\bar{h} \geqslant 6.2$

$$a = 16.1\bar{h}^{-1.54}, \quad b = 47\bar{h}^{-2.94},$$

$$k = b/a, \quad \bar{r}_m = 0.34\bar{h}^{-0.7}.$$

When $\bar{r} = \bar{r}_m$, $\lambda = 0$ the velocity distribution (curve 2, Fig. 1) coincides identically with the fourth degree polynomial

$$u = 2\eta - 2\eta^3 + \eta^4. \quad (2)$$

Comparison of the calculated velocity profiles in the wall layer in the critical point region, from Eq.

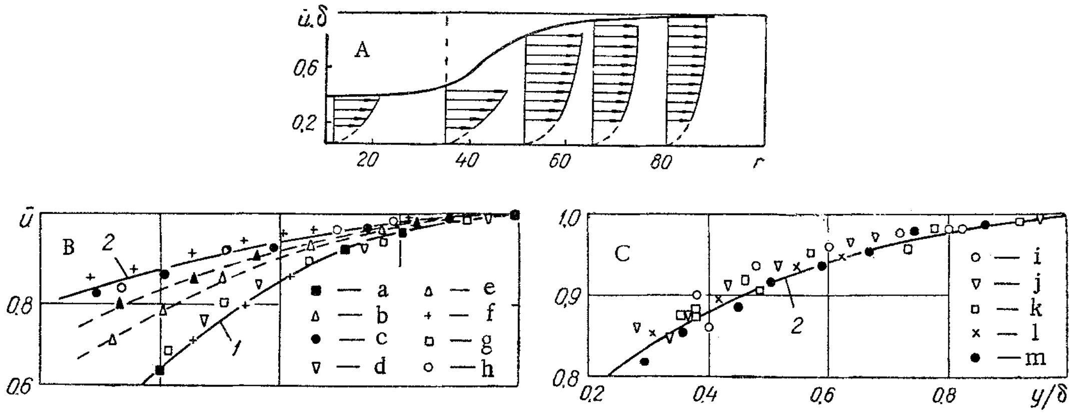

Fig. 2. Variation of velocity profiles $\bar{u}$ and thickness of wall boundary layer δ, mm, over disc radius r, mm, with $d_0 = 31$ mm, $\bar{h} = 2.5$, $u_0 = 15$ m/sec (A), the velocity profiles in the transition region $1 < \bar{r}_* < 2$ (B), and the velocity profiles in the main flow region (C): a, b, c) for $\bar{r}_*$ respectively 1, 1.7, and 2, with $u_0 = 15$ m/sec, $d_0 = 31$ mm, $\bar{h} = 2.5$; d, e, f) $\bar{r}_* = 1$, 1.8 and 2 with $u_0 = 25$ m/sec, $d_0 = 31$ mm, $\bar{h} = 1.6$; g, h) $\bar{r}_* = 1$ and 2 with $u_0 = 25$ m/sec, $d_0 = 31$ mm, $\bar{h} = 8$; i, j, k) $\bar{r}_* = 2$, 2.7, 3.3 (data of [5]) with $u_0 = 19.6$ m/sec, $d_0 = 40$ mm, $\bar{h} = 3.75$; l, m) $\bar{r}_* = 2$ and 3 (authors' data) with $u_0 = 25$ m/sec, $d_0 = 31$ mm, $\bar{h} = 2.5$; 1) calculated from Eq. (2); 2) from Eq. (4).

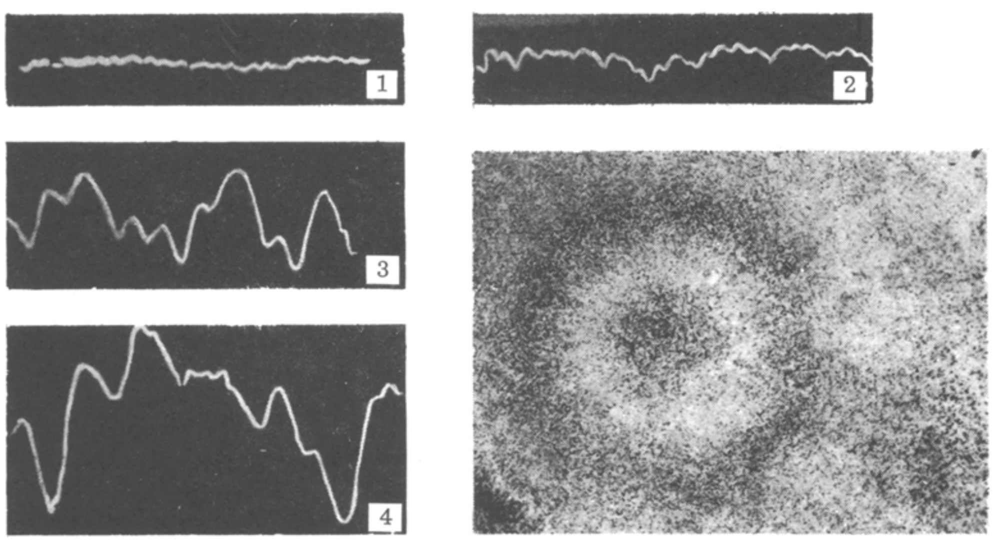

Fig. 3. Oscillograms of velocity fluctuations in the wall boundary layer, and a qualitative picture of the turbulent transition of the laminar boundary layer.

(1), with the experimental data (Fig. 1) shows that the agreement is good.

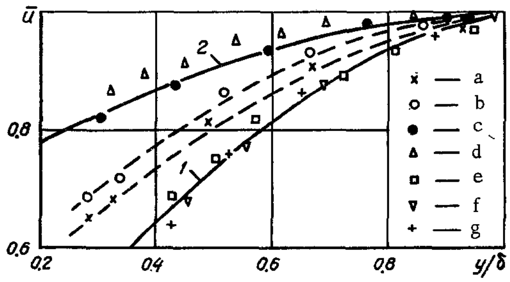

Fig. 4. Velocity profiles in the main flow region with varying Re_0 for d_0 = 31 mm $\bar{r}_*$ = = 2 (a–for Re_0 = 13 600; b–18 000; c–26 000; d–45 000; e–9000) and for $\bar{r}_*$ = 4 (f–Re_0 = = 7000, g–9000; 1) calculated from Eq. (2); 2) from Eq. (3).

The temperature distribution in the wall layer may be represented by a fifth degree polynomial (curve 1 of Fig. 1)

$$\bar{t} = 1 - 2.5y/\delta_t + 5y^3/\delta_t^3 - 5y^4/\delta_t^4 + 1.5y^5/\delta_t^5. \quad (3)$$

In the transition region, which exists in the range $\bar{r}_m \leq 2\bar{r}_m$ or $1 \leq \bar{r}_* \leq 2$, the picture of the flow in the boundary layer changes sharply. While accelerated motion of the fluid particles is observed in the critical point region, in the transition region the velocity u_s at the outer edge of the wall layer begins to drop slowly, the motion of the fluid particles slows down, and their tendency to mix increases.

Figure 2A shows the velocity profiles and the variation of wall layer thickness over the disc radius. The sharp increase in boundary layer thickness and the marked change in velocity distribution are evidence that the laminar boundary layer has become turbulent. The variation in the dimensionless velocity profiles (Fig. 2B), which were obtained for various relative distances $\bar{r}$ and $\bar{h}$, confirms the unstable nature of the motion in the transition region.

In the main flow region the boundary layer (Fig. 2C), according to our measurements and to the experimental data of Schrader [5], is turbulent, the velocity profiles are similar, and they may be described by the one-seventh power law, i.e., by the law of zero-gradient turbulent flow in the boundary layer

$$u/u_s = (y/\delta)^{1/7} \quad (4)$$

The turbulent transition of the laminar boundary layer may also be judged from the velocity fluctuations in the boundary layer, which were recorded with the aid of a hot-wire anemometer on oscillograms. The anemometer wire was located at a distance 0.2–0.3 mm from the surface, while the nozzle was moved normal to the disc surface. In the tests the parameter Re_0 was varied from 20 000 to 150 000, and the relative distance h = 1–10. The oscillograms (Fig. 3) show that the amplitude of the velocity fluctuations in the critical point region are small (first oscillogram). In the transition region they increase sharply (oscillograms 2, 3 and 4), indicating that the laminar boundary layer has become turbulent.

The point of transition can be made visible using the Nickel method [6]. A powdered mixture of kerosene and soot was spread on a white paper surface. After an air jet had been blown at the surface for a short time (Re_0 = 30 000), the qualitative picture shown in Fig. 3 was obtained. The point of transition has the shape of an annulus of darker color and lies in the transition region.

However, turbulent transition of the laminar boundary layer may not occur. Figure 4 shows velocity profiles in the main flow region when Re_0 varies.

It may be seen from Fig. 4 that as Re_0 decreases, the velocity profiles vary, and with Re_0 < 9000 only laminar velocity profiles, fitting a fourth degree polynomial, are observed along the disc radius.

The velocity profiles may be given the form of a dependence of $Re_{\delta *}$ on r_*. The broken line in Fig. 5 shows the start of transition, i.e., it is the curve of critical values of the points of loss of stability of the boundary layer. With decrease of Re_0 the point of loss of stability shifts to the right, to the value $\bar{r}_* = 2$. The laminar boundary layer region is located to the left of the broken line. The least value of $Re_{\delta *}$ at which only a laminar boundary layer is found over the whole disc surface may be called the stability limit in zero-gradient flow. It is $Re_{\delta *}$ = 90. The parameter Re_0 may also serve as a criterion, since the parameter $Re_{\delta *}$ is proportional to $\sqrt{u_s r/\nu}$. If we take into account that $u_s r/\nu$ varies only up to the value $\bar{r}_* = 2$, while for $\bar{r}_* \geq 2$ it remains constant, since $u_m = 1.32\, u_0 d_0/r$ [2, 7] and $Re_r = 1.32\, Re_0$, then $Re_{\delta *} \sim \sqrt{Re_0}$.

Therefore, for $Re_{\delta *} \leq 90$ or $Re_0 \leq 9000$, only a laminar boundary layer exists over the whole surface of the disc.

It may be seen from Fig. 5 what influence a negative pressure gradient has on boundary layer stability.

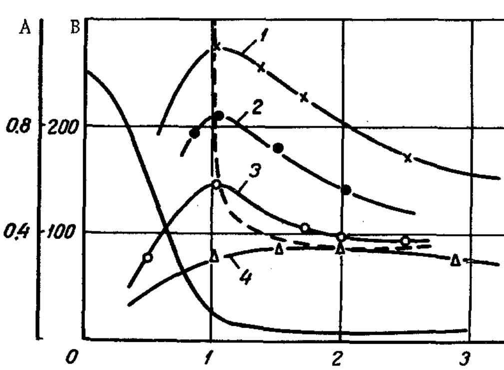

Fig. 5. Dependence of $Re_{\delta *}$ and $(p - p_0)/(\rho u_i^2/2)$ on $\bar{r}_*$ for Re_0 = 50 000 (1), 35 000 (2), 26 000 (3), 9000 (4), $A \equiv (p - p_0)/(\rho u_i^2/2)$; $B \equiv u_m \delta^*/\nu$.

With increase of negative pressure gradient the critical value of the stability limit increases sharply. For $Re_{\delta *}$ > 150 the displacement of the stability loss point to the left is small and we may consider that at this condition in the critical point region the boundary layer is laminar, and, as has been shown above, for $1 \leq \bar{r}_* \leq 2$.

NOTATION

d_0 is the nozzle diameter; $\bar{r}_m = r_m/d_0$ is the relative distance from disc center to point corresponding to maximum axial velocity; $\bar{r} = r/d_0$ is the current disc radius referred to the nozzle diameter; $\bar{r}_* = r/r_m$ is the current disc radius referred to distance from disc center to point corresponding to maximum axial velocity; $\bar{h} = h/d_0$ is the relative distance from nozzle to disc surface; u_0 is the nozzle exit velocity; u_s is the velocity at outer edge of wall boundary layer; $\delta^* = \int_0^\delta \left(1 - \frac{u}{u_s}\right) dy$ is the displacement thickness; $Re_0 = u_0 d_0/\nu$ is the Reynolds number based on initial jet parameters; $Re_r = u_s r/\nu$; $Re_{\delta*} = \delta^* r/\nu$.

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